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Controlled Dense Coding with Symmetric State

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Abstract Two schemes, introducing the projective operator and the auxiliary qubit respectively, for controlled dense coding are investigated by using a three-qubit symmetric state with entanglement, where the supervisor (Cliff) can control an average amount of information transmitted from the sender (Alice) to the receiver (Bob) by adjusting the measurement angle θ. We show that the results for the average amounts of information are unique from the different two schemes. The schemes may be extended to many-qubit systems.

Keywords Controlled dense coding · POVM · Average amount of information

1 Introduction

Quantum entanglement is a quintessential property of quantum mechanics that sets it apart from any classically physical theory. An important feature of entanglement is that it gives rise to correlations that cannot be explained by any locally realistic description of quantum mechanics. In recent years, quantum entanglement has become an important physical resource for quantum communication and quantum computation [1–3]. Dense coding [1] and teleportation [2] are the exhibitions of entanglement in quantum communication. Since advocated by Bennet et al., quantum teleportation has attracted much attention from both experimentalists [4, 5] and theorists [6, 7]. The controlled teleportation protocol was firstly presented in 1998 [8]. In this scheme, an arbitrary single-qubit state may be teleported to either one of two receivers by using a Greenberger-Horne-Zeilinger (GHZ) state, but only one of them can reconstruct the qubit state with the help of the other. Indeed any particle pair in the GHZ state after tracing out the rest of the particles is maximally mixed. This is

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similar to quantum state sharing [9–17], whose basic idea is to let several receivers share an unknown quantum state under cooperations. In general, almost schemes of the quantum state sharing may be used to control teleportation with or without a little modification, and vice versa [18–20].

Werner proved that, under the condition of tightness and with the maximally entangled states, the quantum teleportation and dense coding were one-to-one correspondence, which means that any teleportation scheme works as a dense coding scheme and vice versa [21]. Therefore, the dense coding has been also widely studied in various directions, such as the case of the continuous variables [22, 23], multipartite communication and so on [24–27]. In 2001, Hao et al. presented a controlled dense coding scheme using the GHZ state, where the sender (Alice) could send information to the receiver (Bob) [28]. The quantum channel between Alice and Bob is controlled by supervisor (Cliff) via measurement. This scheme has been realized in experiments [29, 30]. Recently, Chen and Kuang generalized the controlled dense coding protocol of the three-particle GHZ quantum channel to the case of a $(N+2)$-particle GHZ quantum channel via a series of local measurements [31]. Over past decade, the three-qubit entangled states have been investigated in their applications to quantum teleportation and cloning, which have been shown to have certain advantages over the two-particle Bell states. Moreover, it is also common in atomic and molecular physics to consider the cases of the three particles and their resonances. Therefore, it is interesting to investigate three-qubit symmetric state for the dense coding [32].

In this paper, two methods are showed to realize controlled dense coding with three-particle entangled state, which is symmetric under permutation of two qubits and contains three real independent parameters [33]. One of our strategies is to apply partial dependence on the symmetric state on a positive operator valued measure (POVM) [34–36] to achieve dense coding. The second strategy is to perform a Von Neumann measurement by using the non-maximally entangled symmetric state and introducing an auxiliary qubit.

2 Three-Qubit Symmetric State

Let us consider that the three qubits 1, 2 and 3, held by the sender Alice, the receiver Bob and the supervisor Cliff respectively, are in a three-qubit symmetric state under permutation of two qubits,

$$|\Psi\rangle_{123} = \alpha|000\rangle_{123} + \beta|111\rangle_{123} + \gamma|001\rangle_{123} + \delta|110\rangle_{123}, \tag{1}$$

where, without loss of generality, the coefficients α, β, γ and δ are supposed to be real and $\alpha \geq \beta \geq \gamma \geq \delta$. It is noted that normalized condition $\alpha^2 + \delta^2 + \gamma^2 + \delta^2 = 1$ should be satisfied in (1). Under the case of $\alpha = \delta = \gamma = \delta = 1/4$, $|\Psi\rangle_{123}$ is a generalized GHZ state [32].

In order to control the amount of information transmitted from Alice to Bob in the quantum channel with entanglement, after performing a von Neumann measurement on his qubit under the basis,

$$|+\rangle_3 = \cos\theta|0\rangle_3 + \sin\theta|1\rangle_3, \qquad |-\rangle_3 = \sin\theta|0\rangle_3 - \cos\theta|1\rangle_3, \tag{2}$$

where θ is a measured angle under the region $[0, \pi/4]$, Cliff informs his measurement result to Alice and Bob. It is noted that the von Neumann measurement of qubit 3 gives the outcome $|+\rangle_3 >$ or $|-\rangle_3 >$ with equal probability. Thus, the symmetric state (1) in the new basis $\{|+\rangle_3, |-\rangle_3\}$ may be rewritten as

$$|\Psi\rangle_{123} = |\psi\rangle_{12}|+\rangle_3 + |\varphi\rangle_{12}|-\rangle_3, \tag{3}$$

where

$$\begin{aligned} |\psi\rangle_{12} &= (\alpha\cos\theta + \gamma\sin\theta)|00\rangle_{12} + (\delta\cos\theta + \beta\sin\theta)|11\rangle_{12}, \\ |\varphi\rangle_{12} &= (\alpha\sin\theta - \gamma\cos\theta)|00\rangle_{12} + (\delta\sin\theta - \beta\cos\theta)|11\rangle_{12}, \end{aligned} \tag{4}$$

are unnormalized state vectors and their norms stand for the absolute probabilities for each case. Corresponding to Cliff's measurement result $|+\rangle_3$ or $|-\rangle_3$, it is obvious that the state of qubits 1 and 2 collapses to $|\psi\rangle_{12}$ or $|\varphi\rangle_{12}$, respectively. Thus, the non-maximally entangled states, $|\psi\rangle_{12}$ and $|\varphi\rangle_{12}$, with the success probability of dense coding are less than two.

There exist two schemes of dense coding with the symmetric state (1) that are shown in the following.

3 Probabilistic Dense Coding with Projective Operators

At first, we consider the case in which Cliff's measurement result is $|+\rangle_3$ and the state of qubits 1 and 2 collapses to $|\psi\rangle_{12}$, the other can be deduced lately. Generally, the state $|\psi\rangle_{12}$ is not maximally entangled, so the success probability of dense coding for the unnormalized state is less than two.

After receiving the measurement result, Alice uses any one of the four unitary operators $\{I, \sigma_X, i\sigma_Y, \sigma_Z\}$ to operate the shared state $|\psi\rangle_{12}$. Such as

$$\begin{aligned} (I\otimes I)|\psi\rangle_{12} &= (\alpha\cos\theta + \gamma\sin\theta)|00\rangle_{12} + (\delta\cos\theta + \beta\sin\theta)|11\rangle_{12} = |\phi_1\rangle_{12}, \\ (\sigma_X\otimes I)|\psi\rangle_{12} &= (\delta\cos\theta + \beta\sin\theta)|01\rangle_{12} + (\alpha\cos\theta + \gamma\sin\theta)|10\rangle_{12} = |\phi_2\rangle_{12}, \\ (i\sigma_Y\otimes I)|\psi\rangle_{12} &= (\delta\cos\theta + \beta\sin\theta)|01\rangle_{12} - (\alpha\cos\theta + \gamma\sin\theta)|10\rangle_{12} = |\phi_3\rangle_{12}, \\ (\sigma_Z\otimes I)|\psi\rangle_{12} &= (\alpha\cos\theta + \gamma\sin\theta)|00\rangle_{12} - (\delta\cos\theta + \beta\sin\theta)|11\rangle_{12} = |\phi_4\rangle_{12}. \end{aligned} \tag{5}$$

Then Alice sends qubit 1 to Bob, and now Bob has at his disposal two qubits which could be in any one of the four possible states $\{|\phi_1\rangle_{12}, |\phi_2\rangle_{12}, |\phi_3\rangle_{12}, |\phi_4\rangle_{12}\}$. If Bob is able to distinguish all the four nonorthogonal states conclusively, he can extract two classical bits of information. However, the above four states are not mutually orthogonal. According to quantum theory, even though these four non-orthogonal states cannot be distinguished with certainty, it is known that a set of non-orthogonal states which are linearly independent may be distinguished with some probability of success [33, 37, 38]. Fortunately, it is easy to find that the above set $\{|\phi_1\rangle_{12}, |\phi_2\rangle_{12}, |\phi_3\rangle_{12}, |\phi_4\rangle_{12}\}$ is actually linearly independent. Therefore Bob can conclusively distinguish these states with some probability of success.

To distinguish the above set $\{|\phi_1\rangle_{12}, |\phi_2\rangle_{12}, |\phi_3\rangle_{12}, |\phi_4\rangle_{12}\}$, Bob firstly performs a projection onto the subspaces spanned by the basis states $\{|00\rangle, |11\rangle\}$ and $\{|01\rangle, |10\rangle\}$ with corresponding projective operators $P_1 = |00\rangle\langle 00| + |11\rangle\langle 11|$ and $P_2 = |01\rangle\langle 01| + |10\rangle\langle 10|$, respectively. Obviously, P_1 and P_2 are mutually orthogonal, and Bob can discriminate the two subsets of Alice's operators: $\{I, \sigma_Z\}$ and $\{\sigma_X, i\sigma_Y\}$. If Bob obtains P_1, then he knows that the state will be either $|\phi_1\rangle_{12}$ or $|\phi_4\rangle_{12}$. Similarly, if he obtains P_2, the state will be either $|\phi_2\rangle_{12}$ or $|\phi_3\rangle_{12}$. After this projective measurement he gets 1 bit of information [28]. Suppose Bob obtains P_1, then he can perform a generalized measurement on his two qubit states. In the case, A POVM in the subspace $\{|00\rangle, |11\rangle\}$ is [34–36]

$$M_1 = \frac{1}{2}\begin{pmatrix} C^2 & C \\ C & 1 \end{pmatrix}, \qquad M_2 = \frac{1}{2}\begin{pmatrix} C^2 & -C \\ -C & 1 \end{pmatrix}, \qquad M_3 = \begin{pmatrix} 1-C^2 & 0 \\ 0 & 0 \end{pmatrix}. \tag{6}$$

where $C = \frac{\delta\cos\theta+\beta\sin\theta}{\alpha\cos\theta+\gamma\sin\theta}$ and the condition $M_1 + M_2 + M_3 = I$ is satisfied. The POVM has three outcomes, which are independent of the state in the measured system. Therefore, the POVM provides the most generally physically realized measurement in quantum mechanics.

If Bob gets M_1 then the state is $|\phi_1\rangle_{12}$, if he gets M_2 then the state is $|\phi_4\rangle_{12}$, and if he gets M_3 the state is completely indecisive. The absolute success probability of distinguish $|\phi_1\rangle_{12}$ and $|\phi_4\rangle_{12}$ is $2(\delta\cos\theta + \beta\sin\theta)^2$, which is also the probability that Bob obtains another bit of information. Similar procedure may be applied for the case of P_2, one may show that the relevant POVM elements and the success probability are the same. In the case of $C = 1$, corresponding to the maximally entangled state, the absolute success probability becomes two.

If Cliff's measurement result is $|-\rangle_3$, then the state of qubits 1 and 2 collapses to $|\varphi\rangle_{12}$. The situation is more complicated than the former. After Alice's encoding with the four operators $\{I, \sigma_X, i\sigma_Y, \sigma_Z\}$, the state $|\varphi\rangle_{12}$ undergoes one of the following transformations, such as

$$\begin{aligned}
(I\otimes I)|\varphi\rangle_{12} &= (\alpha\sin\theta - \gamma\cos\theta)|00\rangle_{12} + (\delta\sin\theta - \beta\cos\theta)|11\rangle_{12} = |\phi_1\rangle_{12},\\
(\sigma_X\otimes I)|\varphi\rangle_{12} &= (\delta\sin\theta - \beta\cos\theta)|01\rangle_{12} + (\alpha\sin\theta - \gamma\cos\theta)|10\rangle_{12} = |\phi_2\rangle_{12},\\
(i\sigma_Y\otimes I)|\varphi\rangle_{12} &= (\delta\sin\theta - \beta\cos\theta)|01\rangle_{12} - (\alpha\sin\theta - \gamma\cos\theta)|10\rangle_{12} = |\phi_3\rangle_{12},\\
(\sigma_Z\otimes I)|\varphi\rangle_{12} &= (\alpha\sin\theta - \gamma\cos\theta)|00\rangle_{12} - (\delta\sin\theta - \beta\cos\theta)|11\rangle_{12} = |\phi_4\rangle_{12}.
\end{aligned} \tag{7}$$

In this case, the outcomes depend on the relative size of θ and the coefficients of the symmetric state. If $(\alpha\sin\theta - \gamma\cos\theta) \le (\delta\sin\theta - \beta\cos\theta)$, the POVM set with three elements may be expressed as

$$M_1' = \frac{1}{2}\begin{pmatrix} 1 & C' \\ C' & C'^2 \end{pmatrix}, \qquad M_2' = \frac{1}{2}\begin{pmatrix} 1 & -C' \\ -C' & C'^2 \end{pmatrix}, \qquad M_3' = \begin{pmatrix} 0 & 0 \\ 0 & 1-C'^2 \end{pmatrix}. \tag{8}$$

where $C' = \frac{\alpha\sin\theta-\gamma\cos\theta}{\delta\sin\theta-\beta\cos\theta}$. In this case, Bob can discriminate $|\phi_1\rangle_{12}$ from $|\phi_4\rangle_{12}$ (or $|\phi_2\rangle_{12}$ from $|\phi_3\rangle_{12}$) with absolute success probability $2(\alpha\sin\theta - \gamma\cos\theta)^2$. Under the condition of $(\alpha\sin\theta - \gamma\cos\theta) \ge (\delta\sin\theta - \beta\cos\theta)$, similarly, the POVM set with three elements is written as

$$M_1'' = \frac{1}{2}\begin{pmatrix} C''^2 & C'' \\ C'' & 1 \end{pmatrix}, \qquad M_2'' = \frac{1}{2}\begin{pmatrix} C''^2 & -C'' \\ -C'' & 1 \end{pmatrix}, \qquad M_3'' = \begin{pmatrix} 1-C''^2 & 0 \\ 0 & 0 \end{pmatrix}, \tag{9}$$

where $C'' = \frac{\delta\sin\theta-\beta\cos\theta}{\alpha\sin\theta-\gamma\cos\theta}$, and Bob can discriminate the two states in the subspace $\{|00\rangle, |11\rangle\}$ and $\{|01\rangle, |10\rangle\}$ with the same absolute probability $2(\delta\sin\theta - \beta\cos\theta)^2$.

Synthesizing all measurement cases, the average amount of information transmitted from Alice to Bob is a summation of the absolute success probability in the two POVM measurements and can be expressed as

$$I = \begin{cases} 1 + 2(\beta\sin\theta + \delta\cos\theta)^2 + 2(\gamma\cos\theta - \alpha\sin\theta)^2, \\ \quad (\alpha\sin\theta - \gamma\cos\theta) \le (\delta\sin\theta - \beta\cos\theta), \\ 1 + 2(\beta^2 + \delta^2), \\ \quad (\alpha\sin\theta - \gamma\cos\theta) \ge (\delta\sin\theta - \beta\cos\theta). \end{cases} \tag{10}$$

From (10), we see that, under the different condition, the average amount of information transmitted from Alice to Bob is different. Especially, under the condition of $(\alpha\sin\theta - \gamma\cos\theta) \geq (\delta\sin\theta - \beta\cos\theta)$, the average amount of transmitted information is independent of the measured angle. It is known that the condition depends on choosing the measured angle. Therefore, it is helpful for Cliff to control the average amount of information transmitted from Alice to Bob by adjusting the measured angle.

4 Probabilistic Dense Coding with Auxiliary Qubit

Now we discuss that Cliff's measurement result is $|+\rangle_3$. In this case, the state of qubits 1 and 2 collapses to $|\psi\rangle_{12}$. After receiving the measurement result, Alice shares the generally entangled state $|\psi\rangle_{12}$ with Bob who does not know it. Now Alice take a new way to study the non-maximally entangled state by introducing an auxiliary qubit with original state $|0\rangle_{aux}$. Thus a unitary transformation is performed by

$$U_1 = \begin{pmatrix} C & 0 & \sqrt{1-C^2} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \sqrt{1-C^2} & 0 & -C & 0 \end{pmatrix}, \tag{11}$$

on the auxiliary qubit and qubit 1 under the basis $\{|0\rangle_{aux}|0\rangle_1, |0\rangle_{aux}|1\rangle_1, |1\rangle_{aux}|0\rangle_1, |1\rangle_{aux}|1\rangle_1\}$. It is noted that C in (11) is the same as (6). Moreover, If $\alpha = \beta = 1/\sqrt{2}$ and $\gamma = \delta = 0$ are satisfied, (11) will reduce to (6) in Ref. [28]. Therefore, our results are more general than the ones of the three-particle GHZ states. The two-qubit unitary matrix U_1 with the single-qubit identity operation I_2, that is $U_1 \otimes I_2$, transforms the state $|0\rangle_{aux} \otimes |\psi\rangle_{12}$ to

$$\begin{aligned} |\psi\rangle_{aux12} &= \sqrt{2}(\delta\cos\theta + \beta\sin\theta)|0\rangle_{aux}\left[\frac{1}{\sqrt{2}}(|00\rangle_{12} + |11\rangle_{12})\right] \\ &\quad + \sqrt{(\alpha\cos\theta + \gamma\sin\theta)^2 - (\beta\sin\theta + \delta\cos\theta)^2}|1\rangle_{aux}|10\rangle_{12}. \end{aligned} \tag{12}$$

Then Alice performs a von Neumann measurement on the auxiliary qubit under the basis $\{|0\rangle_{aux}, |1\rangle_{aux}\}$. If she obtains $|0\rangle_{aux}$, the qubits 1 and 2 are maximally entangled. Alice now performs one of the four unitary transformations $\{I, \sigma_X, i\sigma_Y, \sigma_Z\}$ on qubit 1 and sends it to Bob. Thus Bob knows he has two qubits in one of the four Bell states resulted from Alice's transformation. By performing a Bell basis measurement, Bob can discriminate Alice's unitary transformation on qubit 1, so 2 bits of information are transmitted. If Alice obtains $|1\rangle_{aux}$, the qubits 1 and 2 are unentangled. Bob can extract only 1 bit of information. Thus, an average number of

$$\begin{aligned} I_1 &= 4(\delta\cos\theta + \beta\sin\theta)^2 + (\alpha\cos\theta + \gamma\sin\theta)^2 - (\beta\sin\theta + \delta\cos\theta)^2 \\ &= 3(\beta^2\sin^2\theta + \delta^2\cos^2\theta) + \alpha^2\cos^2\theta + \gamma^2\sin^2\theta + 2(\alpha\gamma + 3\beta\delta)\cos\theta\sin\theta, \end{aligned} \tag{13}$$

bits of information is transmitted from Alice to Bob.

If Cliff's measurement result is $|-\rangle_3$ under the condition of $(\alpha\sin\theta - \gamma\cos\theta) \leq (\delta\sin\theta - \beta\cos\theta)$, Alice's unitary transformation on the auxiliary qubit and qubit 1 should be changed as

$$U_2' = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & C' & \sqrt{1-C'^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\sqrt{1-C'^2} & C' & 0 \end{pmatrix}, \tag{14}$$

under the basis $\{|0\rangle_{aux}|0\rangle_1, |0\rangle_{aux}|1\rangle_1, |1\rangle_{aux}|0\rangle_1, |1\rangle_{aux}|1\rangle_1\}$. Here C' is the same as the one in (8). Under the operation of the two-qubit unitary transformation $U_2' \otimes I_2$, the state $|0\rangle_{aux} \otimes |\varphi\rangle_{12}$ is transformed as

$$\begin{aligned} |\varphi'\rangle_{aux12} &= \sqrt{2}(\alpha\sin\theta - \gamma\cos\theta)|0\rangle_{aux}\left[\frac{1}{\sqrt{2}}(|00\rangle_{12} + |11\rangle_{12})\right] \\ &\quad + \sqrt{(\delta\sin\theta - \beta\cos\theta)^2 - (\alpha\sin\theta - \gamma\cos\theta)^2}|1\rangle_{aux}|11\rangle_{12}. \end{aligned} \tag{15}$$

Then Alice performs a von Neumann measurement on the auxiliary qubit under the basis $\{|0\rangle_{aux}, |1\rangle_{aux}\}$. If Alice gets the result $|1\rangle_{aux}$, the state of qubits 1 and 2 is unentangled. In this case, the single-bit information can be transmitted. If the measured result is $|0\rangle_{aux}$, the state of qubits 1 and 2 is maximally entangled. The two-bit information can be transmitted. In the case, therefore, Alice can transmit

$$\begin{aligned} I_2' &= 4(\alpha\sin\theta - \gamma\cos\theta)^2 + (\delta\sin\theta - \beta\cos\theta)^2 - (\alpha\sin\theta - \gamma\cos\theta)^2 \\ &= 3(\alpha^2\sin^2\theta + \gamma^2\cos^2\theta) + \delta^2\sin^2\theta + \beta^2\cos^2\theta - 2(3\alpha\gamma + \delta\beta)\cos\theta\sin\theta, \end{aligned} \tag{16}$$

bits of information on average. The average amount of information transmitted from Alice to Bob adds up to

$$I = I_1 + I_2' = 1 + 2(\beta\sin\theta + \delta\cos\theta)^2 + 2(\gamma\cos\theta - \alpha\sin\theta)^2, \tag{17}$$

where the normalized condition is used and I is a function of the measured angle.

Under the case of $(\alpha\sin\theta - \gamma\cos\theta) \geq (\delta\sin\theta - \beta\cos\theta)$, Alice's unitary transformation on the auxiliary qubit and qubit 1 under the same basis is given by

$$U_2'' = \begin{pmatrix} C'' & 0 & \sqrt{1-C''^2} & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \sqrt{1-C''^2} & 0 & -C'' & 0, \end{pmatrix}, \tag{18}$$

where C'' is defined in (9). Under the operation of $U_2'' \otimes I_2$, the state $|0\rangle_{aux} \otimes |\varphi\rangle_{12}$ is transformed to the following state,

$$\begin{aligned} |\varphi''\rangle_{aux12} &= \sqrt{2}(\delta\sin\theta - \beta\cos\theta)|0\rangle_{aux}\left[\frac{1}{\sqrt{2}}(|00\rangle_{12} + |11\rangle_{12})\right] \\ &\quad + \sqrt{(\alpha\sin\theta - \gamma\cos\theta)^2 - (\delta\sin\theta - \beta\cos\theta)^2}|1\rangle_{aux}|10\rangle_{12}. \end{aligned} \tag{19}$$

After Alice's von Neumann measurement under the basis $\{|0\rangle_{aux}, |1\rangle_{aux}\}$, Alice can transmit

$$I_2'' = 3(\delta^2\sin^2\theta + \beta^2\cos^2\theta) + \alpha^2\sin^2\theta + \gamma^2\cos^2\theta - 2(\alpha\gamma + 3\delta\beta)\cos\theta\sin\theta, \tag{20}$$

bits of information on average. The average amount of information transmitted from Alice to Bob is given by

$$I = I_1 + I_2'' = 1 + 2(\delta^2 + \beta^2), \tag{21}$$

which is independent of the measure angle.

Thus, the average amount of information transmitted from Alice to Bob is summarized as

$$I = \begin{cases} 1 + 2(\beta \sin\theta + \delta\cos\theta)^2 + 2(\gamma\cos\theta - \alpha\sin\theta)^2, \\ \quad (\alpha\sin\theta - \gamma\cos\theta) \le (\delta\sin\theta - \beta\cos\theta), \\ 1 + 2(\beta^2 + \delta^2), \\ \quad (\alpha\sin\theta - \gamma\cos\theta) \ge (\delta\sin\theta - \beta\cos\theta). \end{cases} \tag{22}$$

Comparing (10) with (22), we find that the results are the same. Therefore, the two different schemes are equivalent for the controlled dense coding. Their results are unique.

5 Summary

In summary, two schemes, introducing the projective operator and the auxiliary qubit respectively, of realizing controlled dense coding are investigated by using a three-qubit symmetric state with entanglement, where Alice send the information to Bob and Cliff serves as quantum erasure by the measurement. It is proved that the results for the average amounts of information are unique from the different two schemes.

Under the different cases, the average amount of information transmitted from Alice to Bob is different. Therefore, one may obtain more information about the transmission. This is helpful for Cliff to choose some useful measurements for higher success probability.

It is shown that the success probability depends on the measured angle θ. This implies that Cliff may control the average amount of information transmitted from Alice to Bob by adjusting measurement angle.

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